# BRDF Summary

Ahmet Oğuz Akyüz

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**Abstract**

Bidirectional reflectance distribution functions (BRDFs) govern how much a surface reflects lights for every incoming and outgoing light direction. This document summarizes the BRDFs that we will use in the homework. In practice, there is a large number of BRDFs and various good surveys can be found online.

## 1 Introduction

The reflection component of the rendering equation can be written as follows:

$$L_r(x, w_o) = \int_\Omega L_i(x, w_i) f(x, w_i, w_o) \cos(\theta_i) dw_i \quad (1)$$

The terms are shown in Figure 1. Here, $f(x, w_i, w_o)$ represents the BRDF. It tells how much light coming from the $w_i$ direction is reflected along the $w_o$ direction from point $x$. The integral is computed over the entire hemisphere in the normal direction at the point of interest. We will also drop the $x$ parameter as our BRDFs will not be spatially varying.

Note that each BRDF to be explained is comprised of two terms, one for the diffuse component and the other for the specular component. The diffuse components of all BRDFs in this document are the same (except for a difference in normalization). Therefore, the main difference between the BRDFs are due to their different formulization of the specular component.

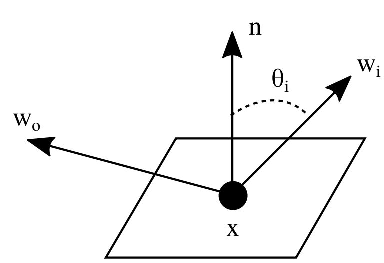


Figure 1: Depiction of the terms used in the rendering equation.

## 2 Phong BRDF

$$f(w_i, w_o) = \begin{cases} k_d + k_s \frac{\cos^p(\alpha_r)}{\cos \theta_i} & \theta_i < 90^o \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Here, $\alpha_r$ is the angle between the perfect reflection direction of $w_i$ across the surface normal and the $w_o$ direction. $p$ is the Phong exponent. Note that it is impossible to normalize the Phong BRDF in its original form so that it becomes energy conserving.

## 3 Modified Phong BRDF

This is basically the Phong BRDF with $\cos(\theta_i)$ removed from the denominator:

$$f(w_i, w_o) = \begin{cases} k_d + k_s \cos^p(\alpha_r) & \theta_i < 90^o \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

## 4 Normalized Modified Phong BRDF

Normalization of a BRDF ensures that it becomes energy conserving. The normalization condition requires that:

$$R(w_i) = \int_\Omega f(w_i, w_o) \cos(\theta_o) dw_o \leq 1 \tag{4}$$

This means that for each incoming direction $w_i$, the total reflectance along all output directions cannot exceed 1. The normalized modified Phong BRDF is equal to:

$$f(w_i, w_o) = \begin{cases} k_d \frac{1}{\pi} + k_s \frac{p+2}{2\pi} \cos^n(\alpha_r) & \theta_i < 90^o \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

## 5 Blinn-Phong BRDF

In this BRDF, we use the angle between the *half vector* and the surface normal instead of the $w_o$ and the perfect reflection direction. The half vector is computed as:

$$w_h = \frac{w_i + w_o}{||w_i + w_o||} \tag{6}$$

$$f(w_i, w_o) = \begin{cases} k_d + k_s \frac{\cos^p(\alpha_h)}{\cos \theta_i} & \theta_i < 90^o \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Here, $\alpha_h$ is the angle between the half vector and the surface normal.

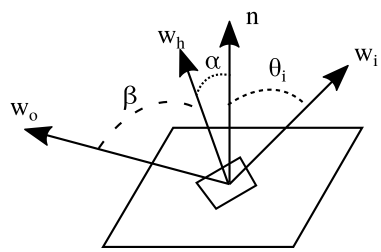


Figure 2: The configuration for the Torrance-Sparrow BRDF.

# 6 Modified Blinn-Phong BRDF

Similar to the modified Phong BRDF, the $\cos(\theta_i)$ term is removed from the denominator:

$$f(w_i, w_o) = \begin{cases} k_d + k_s \cos^p(\alpha_h) & \theta_i < 90^o \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

# 7 Normalized Modified Blinn-Phong BRDF

Despite seeming like magic numbers, the numbers below come from applying the conservation condition (Eq 4) to the modified Blinn-Phong BRDF:

$$f(w_i, w_o) = \begin{cases} k_d \frac{1}{\pi} + k_s \frac{p+8}{8\pi} \cos^n(\alpha_r) & \theta_i < 90^o \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

# 8 Torrance-Sparrow BRDF

This BRDF treats the surface as a collection of micro-facets. Each micro-facet is considered to be a mirror-like reflector exhibiting Fresnel reflection characteristics, represented by the $F$ term. Some of these micro-facets are oriented toward the surface normal and some point away from it. The orientation of micro-facets are represented by a micro-facet distribution function, $D$. Also micro-facets cause shadowing and masking, and this is modeled by a geometry term $G$. The BRDF including all these three terms is defined as:

$$f(w_i, w_o) = \begin{cases} k_d \frac{1}{\pi} + k_s \frac{D(\alpha)F(\beta)G(w_i,w_o)}{4\cos(\theta)cos(\phi)} & \theta_i < 90^o \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

The terms are demonstrated in Figure 2. This BRDF may seem daunting at first, but it is simple evaluate if you follow the following order.

1. Compute the half vector $w_h$ between $w_i$ and $w_o$ as usual.

2. Compute the angle $\alpha$ as $w_h \bullet n$.

3. Compute the probability of this $\alpha$ using $D(\alpha)$. There are various distribution functions used in practice. A commonly used one is Blinn's distribution. Its normalized form is given by:

$$D(\alpha) = \frac{p+2}{2\pi} \cos(\alpha)^p, \tag{11}$$

   where $p$ is the Phong exponent as before.

4. Compute the geometry term $G$ as:

$$G(w_i, w_o) = \min(1, \min(\frac{2(n \bullet w_h)(n \bullet w_o)}{w_o \bullet w_h}, \frac{2(n \bullet w_h)(n \bullet w_i)}{w_o \bullet w_h})) \tag{12}$$

5. Compute the Fresnel reflectance using Schlick's approximation:

$$F(\beta) = R_0 + (1 - R_0)(1 - \cos(\beta))^5, \tag{13}$$

   where $R_0$ indicates the reflectance of the surface for a incoming light ray that is parallel to the surface normal. Remember that this term is computed from the refractive index, $\eta$, of the material as:

$$R_0 = \frac{(\eta - 1)^2}{(\eta + 1)^2} \tag{14}$$